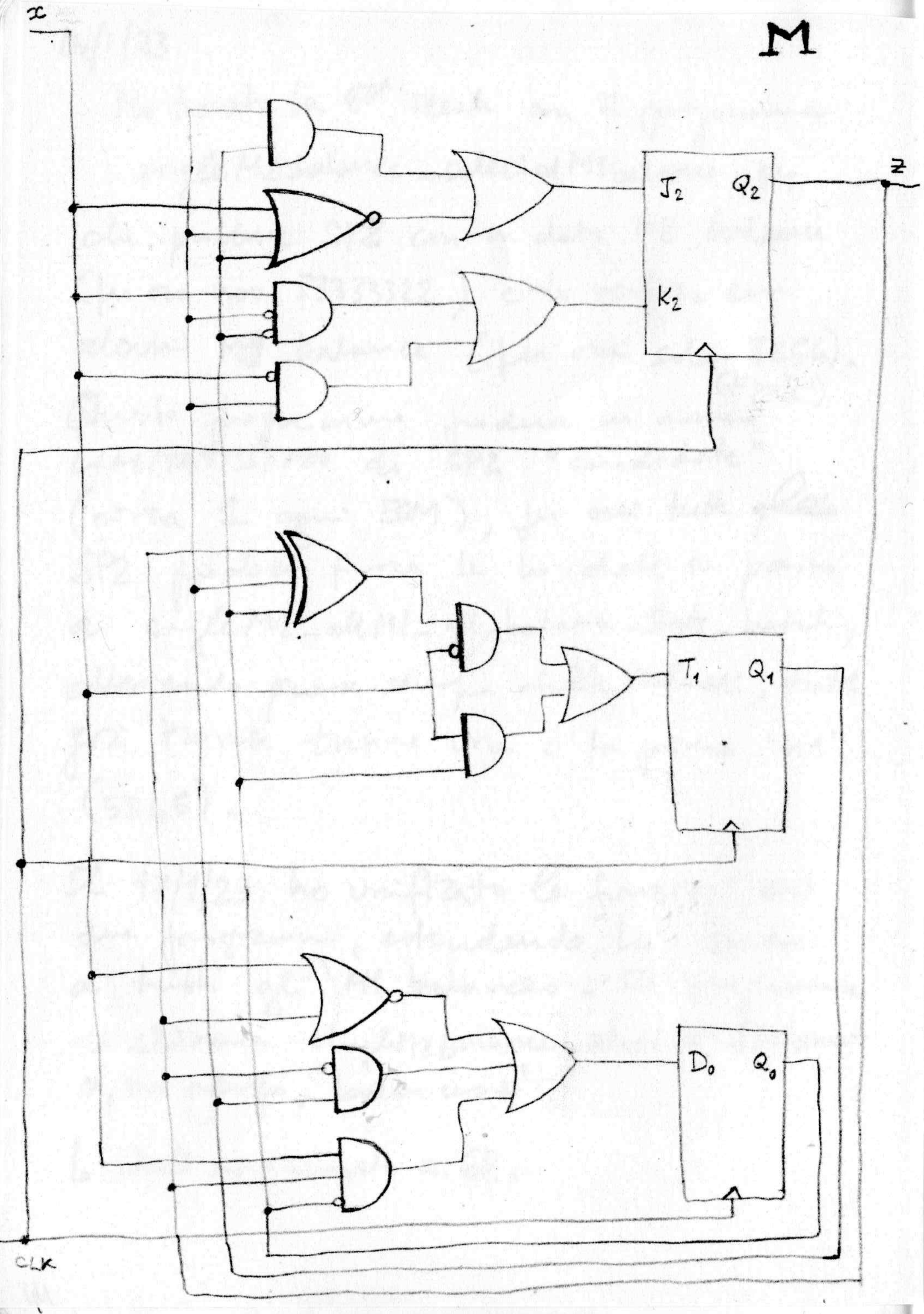


Soluzione del compito del 13/1/23 18/1/23

RETI LOGICHE

Analisi del circuito (macchina M):

$Q'_2 = J_2\overline{Q}_2 + \overline{K}_2 Q_2$ equazione caratteristica del flip-flop JK (*)

$J_2 = \overline{x + Q_1 + Q_0} + Q_1Q_0 = \overline{x}\,\overline{Q}_1\overline{Q}_0 + Q_1Q_0$

$K_2 = x\overline{Q}_1\overline{Q}_0 + \overline{x}Q_1 \Rightarrow$

$\Rightarrow \overline{K}_2 = \overline{x\overline{Q}_1\overline{Q}_0 + \overline{x}Q_1} = (\overline{x\overline{Q}_1\overline{Q}_0})\cdot(\overline{\overline{x}Q_1}) =$

$= (\overline{x} + Q_1 + Q_0)\cdot(x + \overline{Q}_1) =$

$= \overline{x}\,\overline{Q}_1 + xQ_1 + xQ_0 + \overline{Q}_1Q_0$

Sostituendo J_2 e $\overline{K}_2$ in (*):

$Q'_2 = \overline{x}\,\overline{Q}_2\overline{Q}_1\overline{Q}_0 + \overline{Q}_2Q_1Q_0 +$

$+ \overline{x}Q_2\overline{Q}_1 + xQ_2Q_1 + xQ_2Q_0 + Q_2\overline{Q}_1Q_0$

Dunque $Q'_2 = 1$ per le seguenti configurazioni d'ingresso:

$xQ_2Q_1Q_0$ = 0 0 0 0
* 0 1 1 $\{$ 0 0 1 1, 1 0 1 1
0 1 0 * $\{$ 0 1 0 0, 0 1 0 1
1 1 1 * $\{$ 1 1 1 0, 1 1 1 1
1 1 * 1 $\{$ 1 1 0 1, ~~1 1 1 1~~
* 1 0 1 $\{$ ~~0 1 0 1~~, ~~1 1 0 1~~

configurazioni ripetute: si possono eliminare

$xQ_2Q_1Q_0$

0 0 0 0, 0 1 0 0 $\} \rightarrow \overline{x}\,\overline{Q}_1\overline{Q}_0$

0 0 1 1, 1 0 1 1 $\} \rightarrow \overline{Q}_2Q_1Q_0$

0 1 0 1, 1 1 0 1 $\} \rightarrow Q_2\overline{Q}_1Q_0$

1 1 1 0, 1 1 1 1 $\} \rightarrow xQ_2Q_1$

segue:

$Q'_2 = \overline{x}\,\overline{Q}_1\overline{Q}_0 + \overline{Q}_2Q_1Q_0 +$
$+ Q_2\overline{Q}_1Q_0 + xQ_2Q_1$

$Q_1' = T_1\overline{Q}_1 + \overline{T}_1 Q_1$ eq. caratt. del FF T (**)

$T_1 = \overline{x}\,(Q_2 \oplus Q_1 \oplus Q_0) + x Q_0$

$Q_2 Q_1 Q_0$	$Q_2 \oplus Q_1 \oplus Q_0$
0 0 0	0
0 0 1	1
0 1 0	1
0 1 1	0
1 0 0	1
1 0 1	0
1 1 0	0
1 1 1	1

(l'XOR è 1 se le variabili d'ingresso con valore 1 sono in numero DISPARI)

$$\Rightarrow \quad T_1 = \overline{x}\left(\overline{Q}_2\overline{Q}_1 Q_0 + \overline{Q}_2 Q_1 \overline{Q}_0 + Q_2\overline{Q}_1\overline{Q}_0 + Q_2 Q_1 Q_0\right) + x Q_0$$

La funzione $T_1(x, Q_2, Q_1, Q_0)$ è 1 per le seguenti configurazioni d'ingresso:

$x\,Q_2 Q_1 Q_0 =$
0 0 0 1
0 0 1 0
0 1 0 0
0 1 1 1
1 0 0 1
1 0 1 1
1 1 0 1
1 1 1 1
} sono 8

Dunque $\overline{T}_1(x, Q_2, Q_1, Q_0)$ deve valere 1 nelle altre $16-8=8$ configurazioni d'ingresso:

	0 0 0 0	1 0 0 0
→	0 0 1 1	→ 1 0 1 0
	0 1 0 1	1 1 0 0
→	0 1 1 0	→ 1 1 1 0

N.B. solo i mintermini con la → sono $\neq 0$ se moltiplicati per Q_1

Segue $Q_1' = \overline{x}\,\overline{Q}_2\overline{Q}_1 Q_0 + \overline{x}\,Q_2\overline{Q}_1\overline{Q}_0 + x\overline{Q}_1 Q_0$

(**) $+\ \overline{x}\,\overline{Q}_2 Q_1 Q_0 + \overline{x}\,Q_2 Q_1\overline{Q}_0 + x\overline{Q}_2 Q_1\overline{Q}_0 + x Q_2 Q_1\overline{Q}_0$

$= \overline{x}\,\overline{Q}_2 Q_0 + \overline{x}\,Q_2\overline{Q}_0 + x\overline{Q}_1 Q_0 + x Q_1 \overline{Q}_0$

Infine:

$Q_0' = \overline{x + Q_2} + \overline{Q}_2 Q_1 + x Q_2 \overline{Q}_0 =$

$= \overline{x}\,\overline{Q}_2 + \overline{Q}_2 Q_1 + x Q_2 \overline{Q}_0$

Tabella delle verità (transizione di stato):

x	Q_2	Q_1	Q_0	Q_2'	Q_1'	Q_0'
0	0	0	0	1	0	1
0	0	0	1	0	1	1
0	0	1	0	0	0	1
0	0	1	1	1	1	1
0	1	0	0	1	1	0
0	1	0	1	1	0	0
0	1	1	0	0	1	0
0	1	1	1	0	0	0
1	0	0	0	0	0	0
1	0	0	1	0	1	0
1	0	1	0	0	1	1
1	0	1	1	1	0	1
1	1	0	0	0	0	1
1	1	0	1	1	1	0
1	1	1	0	1	1	1
1	1	1	1	1	0	0

tabella verità (versione decimale):

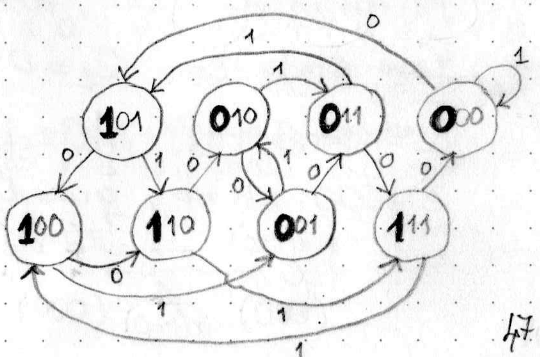
x \ SP	0	1	2	3	4	5	6	7
0	5	3	1	7	6	4	2	0
1	0	2	3	5	1	6	7	4

SF

0	0	0	0	1	1	1	1

Z

Diagramma degli stati

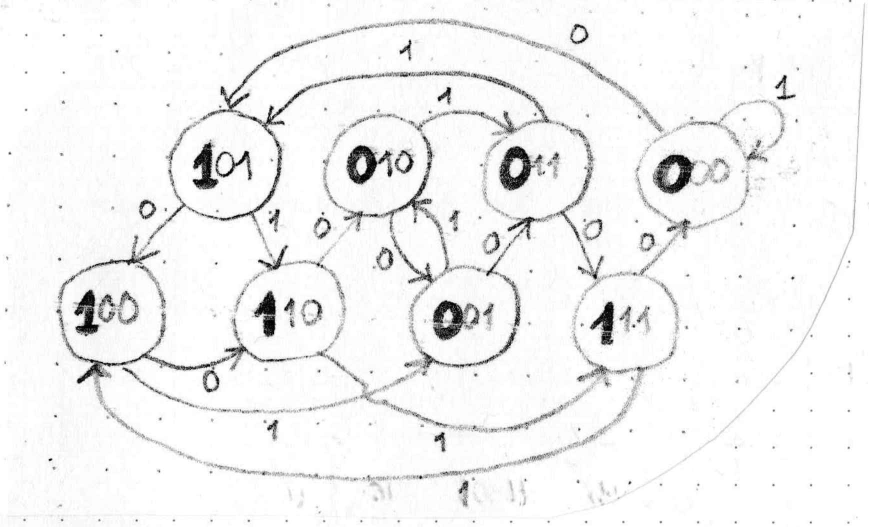


N.B.

$Z = Q_2$

(in grassetto)

Per trovare un paio di sequenze $x(t)$ che fanno produrre alla macchina l'uscita periodica $z(t)$ data, usiamo l'automa:



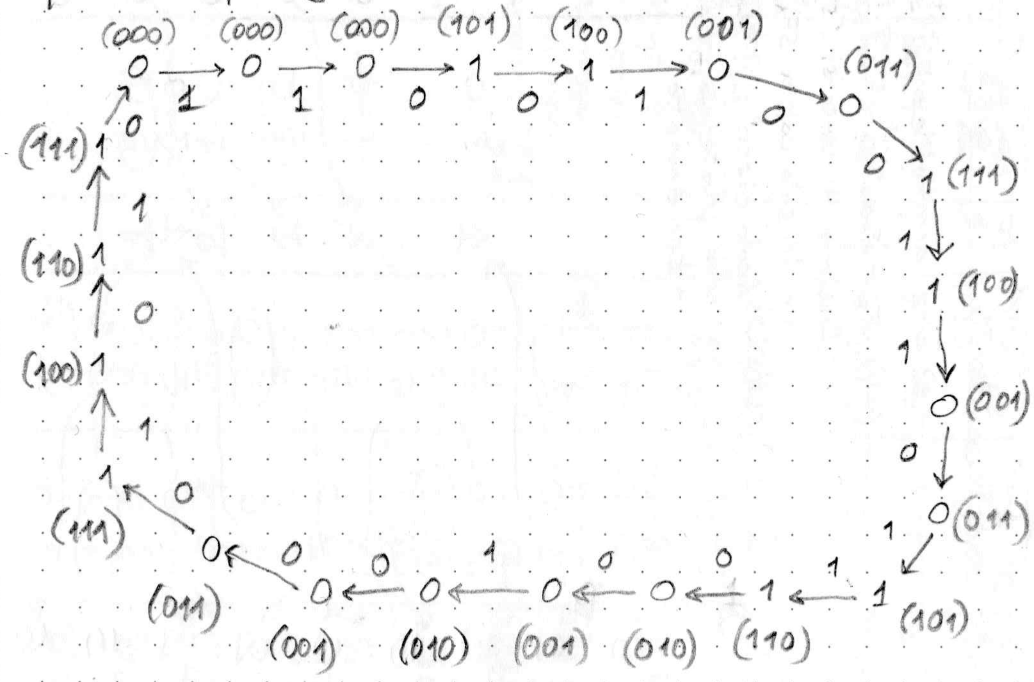
La macchina è inizializzata a ∅; con uscita $Q_2 = 0$. La sequenza di uscita richiesta è 0001100110011000001111

$\uparrow t_0$ $\qquad\qquad$ $\uparrow t_{21}$

Partendo da 000, dobbiamo saltare ad uno stato con $Q_2 = 0$. L'unico stato con questa caratteristica raggiungibile da 000 è 000 stesso, quindi $x = \underline{1}$:

$$\underset{(000)}{\overset{z(t=0)}{0}} \xrightarrow[x(t=0)]{1} \underset{(000)}{\overset{z(t=1)}{0}} \quad \text{etc.}$$

Andando per tentativi, è facile trovare qualche sequenza che chiude il cerchio. Ad es.:



ossia

1 1 0 0 1 0 0 1 1 0 1 1 0 0 1 0 0 0 1 0 1 0

$x(t=0)$ $\qquad\qquad$ $x(t=21)$

Altra sequenza:

1 1 0 1 0 1 1 0 1 0 1 0 1 1 0 1 1 0 1 0 1 0

(N.B. Questo è proprio la FIRST CIRCLE sequence!)

t	z (seq. da realizzare)	z^{next}	SP_A	$x=0$: SF_{A0} z^{next}_{A0}	$x=1$: SF_{A1} z^{next}_{A1}	SP_B	$x=0$: SF_{B0} z^{next}_{B0}	$x=1$: SF_{B1} z^{next}_{B1}	SP_C	$x=0$: SF_{C0} z^{next}_{C0}	$x=1$: SF_{C1} z^{next}_{C1}
0	0	0	**0**	~~5 1~~ (*)	0 0						
1	0	0	**0**	~~5 1~~	0 0						
2	0	1	**0**	5 1	~~0 0~~						
3	1	1	**5**	4 1	6 1						
4	1	0	**4**	~~6 1~~	1 0	**6**	2 0	~~7 1~~			
5	0	0	**1**	3 0	2 0	**2**	1 0	3 0 (**)			
6	0	1	**3**	7 1	5 1	~~2~~ (***)	~~1 0~~	~~3 0~~	~~1~~	~~3 0~~	~~2 0~~
7	1	1	**7**	~~0 0~~	4 1	**5**	4 1	6 1			
8	1	0	**4**	~~6 1~~	1 0	**6**	2 0	~~7 1~~			
9	0	0	**1**	3 0	2 0	**2**	1 0	3 0			
10	0	1	**3**	7 1	5 1	~~2~~	~~1 0~~	~~3 0~~	~~1~~	~~3 0~~	~~2 0~~
11	1	1	**7**	~~0 0~~	4 1	**5**	4 1	6 1			
12	1	0	**4**	~~6 1~~	1 0	**6**	2 0	~~7 1~~			
13	0	0	**1**	3 0	2 0	**2**	1 0	3 0			
14	0	0	~~3~~	~~7 1~~	~~5 1~~	**2**	1 0	3 0	**1**	3 0	2 0
15	0	0	**1**	3 0	2 0	~~3~~	~~7 1~~	~~5 1~~	**2**	1 0	3 0
16	0	0	~~3~~	~~7 1~~	~~5 1~~	**2**	1 0	3 0	**1**	3 0	2 0
17	0	1	~~1~~	~~3 0~~	~~2 0~~	**3**	7 1	5 1	~~2~~	~~1 0~~	~~3 0~~
18	1	1	**7**	~~0 0~~	4 1	**5**	4 1	6 1			
19	1	1	**4**	6 1	~~1 0~~	6	~~2 0~~	7 1			
20	1	1	**6**	~~2 0~~	7 1	7	~~0 0~~	4 1			
21	1	0	**7**	0 0	~~4 1~~	4	~~6 1~~	1 0 (** **)			
22≡0	0	0	**0**	~~5 1~~	0 0	~~1~~	(3 0)	(2 0)			

(*) non accettabile: $z^{next}_{A0} = 1 \neq 0 = z^{next}$

(**) $SF_{B1} = SF_{A0}$, $z^{next}_{B1} = z^{next}_{A0}$: fusione di percorsi (si prosegue con la sola coppia SF_{A0}, z^{next}_{A0})

(***) "vicolo cieco": il percorso non prosegue, e va abbandonato

(** **) percorso che va abbandonato perché non si chiude: $SP_B(22) = 1 \neq 0 = SP_A(0)$

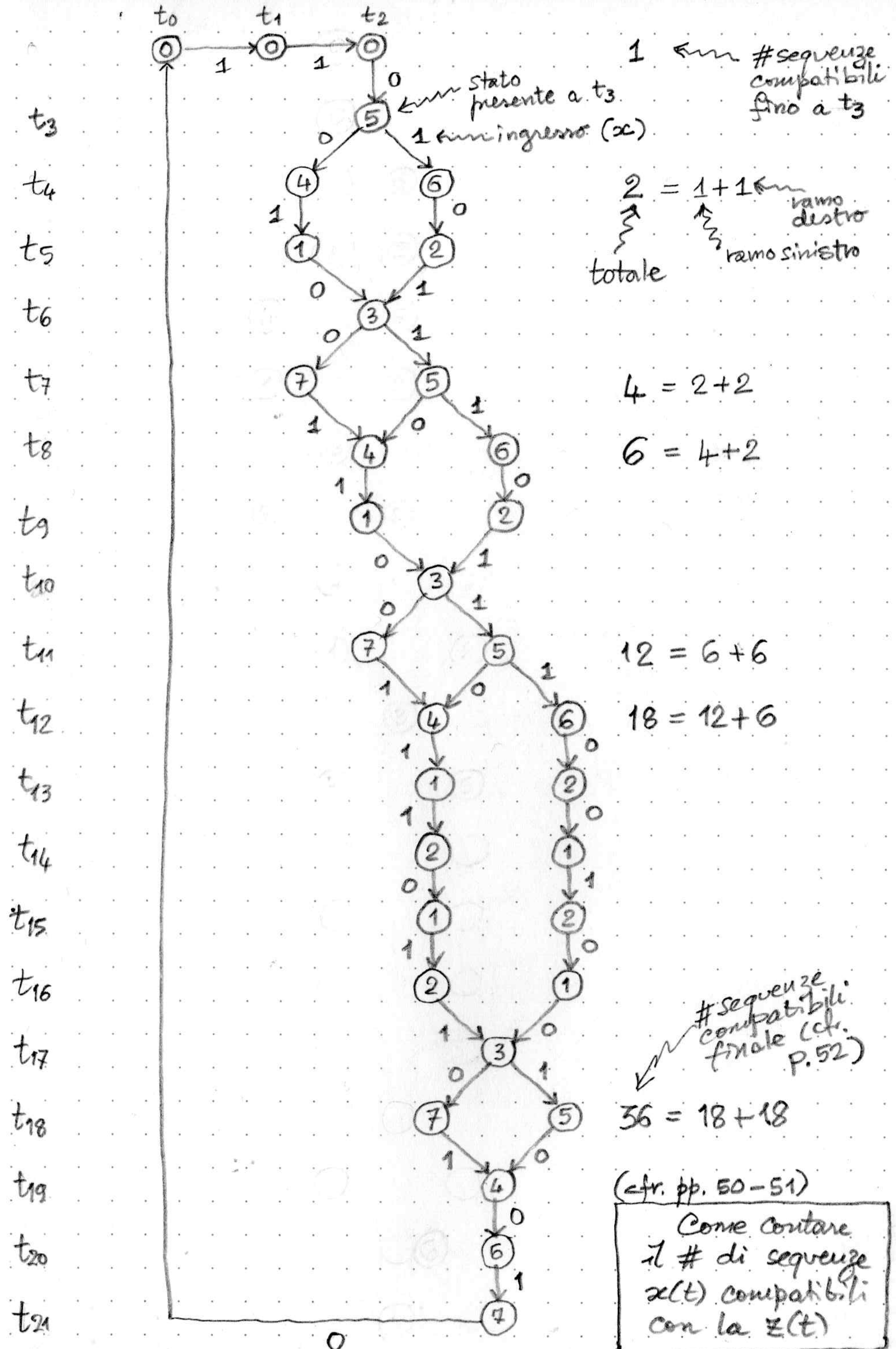
Per il calcolo sistematico dei percorsi si crea l'albero binario completo e si eliminano i percorsi "morti". Per creare l'albero è utile la tabella di p. 47. Il numero complessivo di sequenze $x(t)$ compatibili è **36**. (cfr. pp. 52-3)

Appendice: elenco completo delle sequenze x(t) compatibili verificato con il programma "test_M_completo_2023-01-13.py"

```
target z sequence = [0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1]
machine state transition table SF(x,SP) = [5, 3, 1, 7, 6, 4, 2, 0] (x=0)
                                          [0, 2, 3, 5, 1, 6, 7, 4] (x=1)
                                           0  1  2  3  4  5  6  7  (SP)
(output z = 1 iff SP > 3)

x sequence #01 = [1, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0]
x sequence #02 = [1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0]
x sequence #03 = [1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0]
x sequence #04 = [1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0]
x sequence #05 = [1, 1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0]
x sequence #06 = [1, 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0]
x sequence #07 = [1, 1, 0, 0, 1, 0, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0]
x sequence #08 = [1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0]
x sequence #09 = [1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0]
x sequence #10 = [1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0]
                 (#10 is the First Circle sequence)
x sequence #11 = [1, 1, 0, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0]
x sequence #12 = [1, 1, 0, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0]
x sequence #13 = [1, 1, 0, 0, 1, 0, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0]
x sequence #14 = [1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0]
x sequence #15 = [1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0]
x sequence #16 = [1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0]
x sequence #17 = [1, 1, 0, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0]
x sequence #18 = [1, 1, 0, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0]
x sequence #19 = [1, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 0]
x sequence #20 = [1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 0]
x sequence #21 = [1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 0]
x sequence #22 = [1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 0]
x sequence #23 = [1, 1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 0]
x sequence #24 = [1, 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 0]
x sequence #25 = [1, 1, 0, 0, 1, 0, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0, 1, 0]
x sequence #26 = [1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0, 1, 0]
x sequence #27 = [1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0, 1, 0]
x sequence #28 = [1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0, 1, 0]
x sequence #29 = [1, 1, 0, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0, 1, 0]
x sequence #30 = [1, 1, 0, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0, 1, 0]
x sequence #31 = [1, 1, 0, 0, 1, 0, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0]
x sequence #32 = [1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0]
x sequence #33 = [1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0]
x sequence #34 = [1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0]
x sequence #35 = [1, 1, 0, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0]
x sequence #36 = [1, 1, 0, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0]
```

N.B. La #13 è la sequenza trovata "a mano" (a p. 49)



(cfr. pp. 50–51)

Come contare il # di sequenze x(t) compatibili con la z(t)